# A robot hand-over control scheme for human-like haptic interaction

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Abstract—A robot hand-over control scheme is proposed achieving human-like haptic interaction during object load transfer from a giver to a receiver hand for the planar case. It is assumed that the object has parallel surfaces and unknown mass. The giver initiates the hand-over process while the receiver estimates the transferred object mass adapting its grip force accordingly in a three stage process. The control laws are based on a dynamically stable grasp controller which is modified for the hand-over task. A stable load transfer is securely achieved as shown by the theoretical analysis and illustrated by the simulation results.

#### I. INTRODUCTION

Accomplishment of a natural handing-over task between a human and a robot is at the centre of research interest as robots able to work alongside humans assisting and collaborating with them are becoming increasingly important. Human to human hand-over studies have been conducted in order to reveal a set of key features that can be utilized in robot controllers. In general, humans both as givers and receivers employ a similar strategy for controlling their grip forces in response to changes in load forces during object transfer. It was shown that over the course of the object transfer, as time increased, the grip and load forces decreased linearly for the giver and increased linearly for the receiver except a slightly lower rate near the beginning for the receiver and a much higher rate prior to the end for the giver [1]-[3]. In this way, the giver applies a positive grip even after the load force has reached approximately zero possibly for ensuring safe object transfer while the receiver's role is simply to achieve an efficient load transfer by increasing grip forces given load transfer estimates. The high sensitivity of humans to temporal synchronization indicates the need for the development of human-like robot hand-over control strategies.

This work proposes a handing-over control algorithm which enables robots to perform object hand-overs with safety, efficiency, timing and interaction fluency like humans do. The focus is on the haptic interaction of a giver and a receiver hand. The analysis in confined in the 2D space under the effect of gravity for an object with parallel surfaces but can be extended to objects of unknown shape. A dynamic stable grasp controller is used for each of the two fingered hands and is modified for the hand-over task. Equilibrium and stability analysis is performed and simulation results are given revealing the smooth human-like performance of the proposed control scheme.

## II. PRELIMINARIES

This section introduces the nomenclature and the definitions required for the analysis of the haptic interaction between a giver and a receiver robotic hand. Fig. 1 illustrates the system considered at an instance of the hand-over process. It is consisted of two dual fingered robotic hands in contact with a rigid object of mass  $m_o$  with parallel surfaces and width l in the gravity field. Each hand consists of two 3 degrees of freedom robotic fingers with revolute joints and rigid hemispherical tips of radius r. Vector  $\mathbf{q_i} = \begin{bmatrix} q_{i1} & q_{i2} & q_{i3} \end{bmatrix}^T$  denotes the joint angles for the  $i_{th}$  finger. In the following,  $R_{ab}$  denotes the rotation matrix of frame  $\{b\}$  with reference to frame  $\{a\}$  unless the reference frame is the inertia frame  $\{P\}$  in which case it is omitted.  $R(\theta)$  is a rotation through an angle  $\theta$  about the z axis that is normal to the x-y plane pointing outwards.



Fig. 1: System of robotic fingers grasping a rigid object with parallel surfaces

Let  $\{P\}$  be the inertia frame attached at the base of the first finger and  $\{O\}$  be the object frame placed at its center

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Fig. 2: Finger tip and contact frames

of mass (Fig. 1) and described by the position vector  $\mathbf{p}_{\mathbf{o}} \in \mathbb{R}^2$  and the rotation matrix  $R_o = R(\theta_o)$ .

Let  $\{t_i\}$  be the  $i_{th}$  fingertip frame described by position vector  $\mathbf{p_{t_i}} \in \mathbb{R}^2$  and rotation matrix  $R_{t_i} = R(\phi_i)$ , with  $\phi_i = \sum_{i=1}^{3} q_{ij}$ . Let frame  $\{c_i\}$  be attached at the contact point

of each finger with the object with its x axis aligned with the normal to the object surface pointing inwards and let  ${}^{o}\mathbf{p_{oc_{i}}} = [X_{i} Y_{i}]^{T}$  be its position on the object frame. Let the orientation of  $\{c_{i}\}$  relative to  $\{t_{i}\}$  be described by  $R_{t_{1}c_{1}} =$  $R(\phi_{t_{i}})$  (Fig. 2). Frame  $\{c_{i}\}$  is described by position vector  $\mathbf{p_{c_{i}}} \in \mathbb{R}^{2}$  and rotation matrix  $R_{c_{i}} = R(\phi_{i} + \phi_{t_{i}})$ . Let  $\mathbf{n_{c_{i}}}, \mathbf{t_{c_{i}}} \in \mathbb{R}^{2}$  be the normal pointing inwards and the tangential vectors to the object at the contact points, expressed in  $\{P\}$ , hence  $R_{c_{i}} = [\mathbf{n_{c_{i}}}, \mathbf{t_{c_{i}}}]$ . Notice that  $\mathbf{p_{c_{i}}} = \mathbf{p_{t_{i}}} + r\mathbf{n_{c_{i}}}$ .

We model the system under the following contact and rolling constraints [4]:

$$\begin{bmatrix} D_{ii} & D_{i5} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_i \\ \dot{\mathbf{p}}_o \\ \dot{\theta}_o \end{bmatrix} = 0, \begin{bmatrix} A_{ii} & A_{i5} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_i \\ \dot{\mathbf{p}}_o \\ \dot{\theta}_o \end{bmatrix} = 0 \quad (1)$$

where

$$D_{ii} = \mathbf{n_{c_i}}^T J_{v_i}, \qquad D_{i5} = \begin{bmatrix} -\mathbf{n_{c_i}}^T & \mathbf{n_{c_i}}^T \hat{p}_{oc_i} \end{bmatrix} \quad (2)$$

$$A_{ii} = \mathbf{t_{c_i}}^{I} J_{v_i} + r_i J_{\omega_i}, \quad A_{i5} = \begin{bmatrix} -\mathbf{t_{c_i}}^{I} & \mathbf{t_{c_i}}^{I} \hat{p}_{oc_i} \end{bmatrix}$$
(3)

with  $\mathbf{p}_{\mathbf{oc}_{i}} = \mathbf{p}_{\mathbf{c}_{i}} - \mathbf{p}_{\mathbf{o}}$  and for a vector  $\mathbf{p} = [a \ b]^{T}$  we define  $\hat{\mathbf{p}} = [b \ -a]^{T}$  so that  $\hat{\mathbf{p}}^{T}\mathbf{k} \ \forall k \in \mathbb{R}^{2}$  defines the outer product  $\mathbf{p} \times \mathbf{k}$ . The Jacobian matrices  $J_{v_{i}} = J_{v_{i}}(\mathbf{q}_{i}) \in \mathbb{R}^{2 \times 3}$ ,  $J_{\omega_{i}} = J_{\omega_{i}}(\mathbf{q}_{i}) \in \mathbb{R}^{1 \times 3}$  relate the joint velocity  $\dot{\mathbf{q}}_{i} \in \mathbb{R}^{3}$  with the  $i_{th}$  fingertip linear and rotational velocities  $\dot{\mathbf{p}}_{\mathbf{t}_{i}} \in \mathbb{R}^{2}$  and  $\omega_{\mathbf{t}_{i}} = \dot{\phi}_{i} \in \mathbb{R}$  respectively as follows:

$$\dot{\mathbf{p}}_{\mathbf{t}_{\mathbf{i}}} = J_{v_i} \dot{\mathbf{q}}_{\mathbf{i}} \quad , \quad \omega_{\mathbf{t}_{\mathbf{i}}} = J_{\omega_i} \dot{\mathbf{q}}_{\mathbf{i}} \tag{4}$$

The first equation in (1) is the contact constraint implying that the fingertip cannot penetrate or leave the object's surface. The second equation in (1) is the rolling constraint denoting that the velocity of the contact point on the fingertip surface is equal to the velocity of the contact point on the object surface.

The system dynamics under the contact and rolling constraints (1) on the vertical plane is described by the following equations for both fingers and the object:

$$M_{i}(\mathbf{q}_{i})\ddot{\mathbf{q}}_{i} + C_{i}(\mathbf{q}_{i}, \dot{\mathbf{q}}_{i})\dot{\mathbf{q}}_{i} + \mathbf{g}_{i}(\mathbf{q}_{i}) + D_{ii}{}^{T}f_{i} + A_{ii}{}^{T}\lambda_{i} = \mathbf{u}_{i}$$
(5)

$$M\begin{bmatrix} \ddot{\mathbf{p}}_{o}\\ \ddot{\theta}_{o} \end{bmatrix} + \sum_{i=1}^{4} \left( D_{i5}{}^{T}f_{i} + A_{i5}{}^{T}\lambda_{i} \right) = \begin{bmatrix} 0\\ -m_{o}g\\ 0 \end{bmatrix}$$
(6)

where  $M_i(\mathbf{q_i}) \in \mathbb{R}^{3\times3}, M = diag(M_o, I_o)$ , with  $M_o = diag(m_o, m_o)$  the positive definite inertia matrices of the  $i_{th}$  finger and object respectively and  $m_o$ ,  $I_o$  denote the object's mass and moment of inertia and  $C_i(\mathbf{q_i}, \dot{\mathbf{q_i}})\dot{\mathbf{q_i}} \in \mathbb{R}^3$  the vector of Coriolis and centripetal forces of the  $i_{th}$  finger. Furthermore,  $\mathbf{g_i}(\mathbf{q_i}) \in \mathbb{R}^3$  is the gravity vector, g the gravity acceleration and the Lagrange multipliers  $f_i$  and  $\lambda_i$  represent the applied normal and tangential constraint forces respectively at the contacts. Last,  $\mathbf{u_i} \in \mathbb{R}^3$  is the vector of applied joint torques to the  $i_{th}$  finger.

*Remark*: A rolling constrained model implies that, in practice, contact friction is sufficient to sustain the tangential forces needed for the rolling motion.

A controller achieving stable grasp of the object without utilizing any information on exact contact locations, contact forces and object weight has been proposed in [4] and is given by:

$$\mathbf{u}_{\mathbf{i}} = \mathbf{g}_{\mathbf{i}}(\mathbf{q}_{\mathbf{i}}) - k_{v_{i}}\dot{\mathbf{q}}_{\mathbf{i}} + (-1)^{i+1}\frac{fd}{2r}J_{vi}{}^{T}(\mathbf{p}_{\mathbf{t}_{2}} - \mathbf{p}_{\mathbf{t}_{1}}) - J_{\omega_{i}}{}^{T}r\hat{N}_{i} + \frac{\hat{m}_{o}g}{2}J_{vi}{}^{T}\begin{bmatrix}0\\1\end{bmatrix}$$
(7)

where

$$\hat{N}_{i}(t) = \frac{r}{\gamma_{i}}(\phi_{i}(t) - \phi_{i}(0)),$$
(8)

$$\hat{m}_o(t) = \hat{m}_o(0) - \frac{g}{2\gamma_M} \left( \mathbf{p_{t_1}} + \mathbf{p_{t_2}} \right)^T \begin{bmatrix} 0\\1 \end{bmatrix}, \qquad (9)$$

are the estimations of the tangential forces and object mass respectively with  $k_{v_i}$ ,  $\gamma_i$ ,  $\gamma_M$  being positive constant gains,  $\hat{m}_o(0)$  is an initial guess of the object mass  $m_o$  and  $f_d$  is a positive constant reflecting the desired grasping force. Notice that this control law applies forces at the direction of the line connecting the fingertips (third term) and compensates for the unknown rotational moments developed at the fingertips and the object with the last two terms. After establishing an initial contact with the object, this controller achieves a stable grasp equilibrium by fingertip rolling. For a redundant system (like the two 3 dof fingers) mass estimates converge to the real object mass  $\hat{m}_{o\infty} = m_o$  [4]. Hereafter, the " $\infty$ " in the subscript denotes equilibrium values. At equilibrium the torque balance is achieved at positions satisfying the following equation:

$$f_d (Y_{1\infty} - Y_{2\infty}) + \frac{m_o g \sin \theta_{o\infty}}{2} (Y_{1\infty} + Y_{2\infty}) = 0 \quad (10)$$

which implies finger opposability  $(Y_{1\infty} = Y_{2\infty})$  only when the object is eventually upright  $(\theta_{o\infty} = 0)$  or the contact points are at each side of the center of mass  $(Y_{1\infty} = Y_{2\infty} =$  0). At equilibrium, each contact force compensates for half the object weight and contributes to the grasping force

$$f_{i\infty} = (-1)^{i+1} \frac{f_d}{2r} \mathbf{n_{ci}}^T \left( \mathbf{p_{t_2}} - \mathbf{p_{t_1}} \right) + \frac{m_o g}{2} \mathbf{n_{ci}}^T \begin{bmatrix} 0\\1 \end{bmatrix}, i = 1, 2$$

$$\lambda_{i\infty} = (-1)^{i+1} \frac{f_d}{2r} \mathbf{t_{ci}}^T (\mathbf{p_{t_2}} - \mathbf{p_{t_1}}) + \frac{m_o g}{2} \mathbf{t_{ci}}^T \begin{bmatrix} 0\\1 \end{bmatrix}, i = 1, 2$$

while tangential force estimates converge to equilibrium values  $\hat{N}_{i\infty} = -\lambda_{i\infty}$ . Furthermore, tangential forces at equilibrium satisfy the following equations  $\hat{N}_{1\infty} + \hat{N}_{2\infty} = \frac{f_d}{2r}(Y_{1\infty} - Y_{2\infty})$  and  $\hat{N}_{2\infty} - \hat{N}_{1\infty} = m_o g \cos \theta_{o\infty}$ . In this work, we modify this controller for a giver and

In this work, we modify this controller for a giver and a receiver hand embedding human-like features during a hand-over task for an unknown mass object. The hand-over strategy is described in the following section.

## **III. HAND-OVER STRATEGY**

The hand-over task is analysed into three stages.

**Stage 1**: The giver (fingers i = 1, 2) stably grasps the object using controller (7) given a pre-set constant  $f_d$ . At equilibrium, estimates of the mass in (9) converge to the actual object mass as mentioned above.

**Stage 2:** The receiver (fingers i = 3, 4) controlled by (7) for i = 3, 4, comes into an initial contact with the object at  $t_0$  and applies a desired constant force  $f_d = f_{init}$ . Clearly, in (7) and (9),  $\mathbf{p_{t_1}}$ ,  $\mathbf{p_{t_2}}$  are replaced by  $\mathbf{p_{t_3}}$ ,  $\mathbf{p_{t_4}}$  respectively. The resulting transient on the giver hand may be sensed by force or tactile sensing signalling the possibility for the object load transfer. In general, multi-modal sensorial data (vision, tactile sensing) may be used to monitor Stage 2.

**Stage 3**: The giver initiates the object load transfer, at  $t_{start} > t_0$  and for a pre-set  $\Delta t$  sec duration, utilizing the following human-like pre-set time functions [3] in place of the object mass estimate  $(\hat{m}_o)$  and desired grasping force  $(f_d)$  in (7).

$$m_g(t) = \begin{cases} (m_o - m_{gf}) \left( 1 - \frac{t - t_{start}}{\Delta t} \right) + m_{gf}, t < t_f \\ m_{gf}, t > t_f \end{cases}$$
(11)

$$f_{dg}(t) = \begin{cases} (f_d - f_{dgf}) \left( 1 - \frac{t - t_{start}}{\Delta t} \right) + f_{dgf}, t < t_f \\ f_{dgf}, t > t_f \end{cases}$$
(12)

where  $m_{gf}$ ,  $f_{dgf}$  are the object mass and the desired grasping force of the giver at the end of the object load transfer at  $t_f = t_{start} + \Delta t$ . Equations (11), (12) linearly decrease the load and grip forces. In a human-like strategy,  $m_{gf} = 0$  and  $f_{dgf} \neq 0$  in order to ensure a safe object transfer. In that case, total object release (i.e. setting  $f_{dg} = 0$ ) is signaled by the micro-sliding that is induced when the receiver hand/object withdraws.

The receiver estimates the transferred object mass  $\hat{m}_o$  via (9) and during this stage increases or adapts its grip force  $f_{dr}(t)$  which is utilized in place of  $f_d$  in (6). One could use a constant grasping force for the receiver  $f_{dr}(t) = f_{init}$  in all stages. This, however, is not an efficient control strategy and

in practice may lead to the object slipping from the receiver hand if the grasping force is not adequate to keep the contact forces inside the friction cone. The following law is proposed instead:

$$f_{dr}(t) = f_{init} + \varepsilon \hat{m}_o(t) \tag{13}$$

where  $\varepsilon$  is a positive control constant, that should ideally be chosen or adapted to the frictional properties of the contact.

Notice that the giver's controller switches desired values depending on the hand-over stage and can therefore be considered as the initiator of the hand-over process while the receiver's controller does not involve any changes. The underlying assumption is that the receiver is a fully trusted and responsive agent.

#### A. System Equilibrium

Substituting the modified control law (7)-(9) with (11) - (13) into (5), (6) utilizing (2), (3), the closed loop system can be written in terms of the force and object mass errors as follows:

$$M_{i}\ddot{\mathbf{q}}_{i} + C_{f_{i}}\dot{\mathbf{q}}_{i} + D_{ii}{}^{T}\Delta f_{i} + A_{ii}{}^{T}\Delta\lambda_{i} + rJ_{\omega_{i}}{}^{T}\Delta N_{i} \quad , j = 1,2$$

$$+ (j-1)\Delta M \frac{9}{2} J_{vi}^{T} \begin{bmatrix} 0\\1 \end{bmatrix} = 0$$
(14)

$$M_{o}\ddot{\mathbf{p}}_{o} - \sum_{i=1}^{n} (\mathbf{n}_{c_{i}} \Delta f_{i} + \mathbf{t}_{c_{i}} \Delta \lambda_{i}) = 0$$
(15)

$$I_{o}\ddot{\theta}_{o} + \sum_{i=1}^{3} \hat{p}_{oc_{i}}^{T} (\mathbf{n_{c_{i}}} \Delta f_{i} + \mathbf{t_{c_{i}}} \Delta \lambda_{i}) + S_{N} = 0$$
(16)

where  $C_{f_i} = (C_i + k_{v_i}I_3)$  with  $I_3$  being the identity matrix of dimension 3, j = 1 for the giver, j = 2 for the receiver and defining the released object mass by  $m_r = m_0 - m_g(t)$ as well as denoting

$$m_j = \begin{cases} m_g & , j = 1\\ m_r & , j = 2 \end{cases}$$
(17)

the rest of the terms are given by the following equations

$$\Delta f_i = f_i - (-1)^{i+1} \mathbf{n_{ci}}^T \mathbf{F_j} - \frac{m_j g}{2} \mathbf{n_{ci}}^T \begin{bmatrix} 0\\1 \end{bmatrix}$$
(18)

$$\Delta \lambda_i = \lambda_i - (-1)^{i+1} \mathbf{t_{ci}}^T \mathbf{F_j} - \frac{m_j g}{2} \mathbf{t_{ci}}^T \begin{bmatrix} 0\\1 \end{bmatrix}$$
(19)

$$\Delta N_i = \hat{N}_i + (-1)^{i+1} \mathbf{t_{ci}}^T \mathbf{F_j} + \frac{m_j g}{2} \mathbf{t_{ci}}^T \begin{bmatrix} 0\\1 \end{bmatrix}$$
(20)

$$\Delta M = m_r - \hat{m}_o \tag{21}$$

$$S_{N} = \frac{f_{dg}}{2r} \left( \hat{p}_{oc_{1}}^{T} - \hat{p}_{oc_{2}}^{T} \right) \left( \mathbf{p_{t_{2}}} - \mathbf{p_{t_{1}}} \right) + \frac{m_{gg}}{2} \left( \hat{p}_{oc_{1}}^{T} + \hat{p}_{oc_{2}}^{T} \right) \begin{bmatrix} 0\\1 \end{bmatrix} + \frac{f_{dr}}{2r} \left( \hat{p}_{oc_{3}}^{T} - \hat{p}_{oc_{4}}^{T} \right) \left( \mathbf{p_{t_{4}}} - \mathbf{p_{t_{3}}} \right) + \frac{m_{rg}}{2} \left( \hat{p}_{oc_{3}}^{T} + \hat{p}_{oc_{4}}^{T} \right) \begin{bmatrix} 0\\1 \end{bmatrix}$$
(22)

with

$$\mathbf{F_j} = \begin{cases} \frac{f_{dg}}{2r} \left( \mathbf{p_{t_2}} - \mathbf{p_{t_1}} \right) &, j = 1\\ \frac{f_{dr}}{2r} \left( \mathbf{p_{t_4}} - \mathbf{p_{t_3}} \right) &, j = 2 \end{cases}$$
(23)

In order to find the equilibrium state of the system in the end of the hand-over process, we set velocities and accelerations to zero in (14):

$$D_{ii}^{T}\Delta f_{i} + A_{ii}^{T}\Delta\lambda_{i} + J_{\omega_{i}}^{T}r\Delta N_{i} + (j-1)\Delta M\frac{g}{2}J_{vi}{}^{T}\begin{bmatrix}0\\1\end{bmatrix} = 0$$

which using (2), (3) can be written as:

$$\begin{bmatrix} J_{v_i}^T & J_{\omega_i}^T \end{bmatrix} \begin{bmatrix} \mathbf{n_{ci}} \Delta f_i + \mathbf{t_{ci}} \Delta \lambda_i + (j-1)\Delta M \frac{g}{2} \begin{bmatrix} 0\\ 1 \end{bmatrix} \\ r(\Delta \lambda_i + \Delta N_i) \end{bmatrix} = 0$$

Assuming a full rank Jacobian matrix  $J_i = \begin{bmatrix} J_{v_i}^T & J_{\omega_i}^T \end{bmatrix}$ , we obtain

$$\mathbf{n_{ci}}\Delta f_i + \mathbf{t_{ci}}\Delta\lambda_i + (j-1)\Delta M\frac{g}{2} \begin{bmatrix} 0\\1 \end{bmatrix} = 0 \qquad (24)$$

$$\Delta \lambda_i + \Delta N_i = 0 \tag{25}$$

Adding equations (24) for all *i* and *j* and substituting the object's translational motion equation (15) at equilibrium  $(\ddot{\mathbf{p}}_{\mathbf{o}} = 0)$  yields

$$\Delta M = 0 \tag{26}$$

and  $\mathbf{n_{ci}}\Delta f_i + \mathbf{t_{ci}}\Delta\lambda_i = 0$  that owing to the independent directions leads to:

$$\Delta f_i = \Delta \lambda_i = 0 \tag{27}$$

Consequently, (25) yields

$$\Delta N_i = 0 \tag{28}$$

Given (27), the object's rotational motion equation (16) at equilibrium ( $\ddot{\theta}_{o} = 0$ ) yields a zero rotational torque acting at the object

$$S_N = 0 \tag{29}$$

From (26) - (28), it is clear that all force and object mass errors are zero at equilibrium. Specifically, (26) implies that, at equilibrium, the receiver estimates correctly the released mass of the object while from (28) we conclude that both giver and receiver estimate the actual tangential forces at the fingertips.

Expressing (29) in the object frame, utilizing (22) and the contact constraints, the torque balance achieved yields the following equation regarding contact positions at equilibrium:

$$f_{dgf}(Y_{1\infty} - Y_{2\infty}) + f_{dr\infty}(Y_{3\infty} - Y_{4\infty}) + \frac{g \sin \theta_{o\infty}}{2} \left[ m_{gf}(Y_{1\infty} + Y_{2\infty}) + m_{r\infty}(Y_{3\infty} + Y_{4\infty}) \right] = 0$$
(30)

where  $m_{r\infty} = m_o - m_{gf}$  and  $f_{dr\infty} = f_{init}$  or  $f_{dr\infty} = f_{init} + \varepsilon m_{r\infty}$  if (13) is used for the receiver's grip force

 $f_{dr}(t)$  adaptation. Notice the similarities between this relationship and (10) which holds for one hand grasp.

Furthermore, adding equations (28) for i = 1, 2 and i = 3, 4, expressing the result in the object frame, utilizing the contact constraints and assuming that total release of the object mass is desired ( $m_{qf} = 0, m_{r\infty} = m_o$ ) yields:

$$\hat{N}_{1\infty} = \hat{N}_{2\infty} \tag{31}$$

$$2\hat{N}_{1\infty} = \frac{f_{dgf}}{r} \left( Y_{1\infty} - Y_{2\infty} \right) \tag{32}$$

$$\hat{N}_{3\infty} + \hat{N}_{4\infty} = \frac{f_{init} + \varepsilon m_o}{r} \left( Y_{3\infty} - Y_{4\infty} \right)$$
(33)

$$\hat{N}_{4\infty} - \hat{N}_{3\infty} = m_o g \cos \theta_o \tag{34}$$

while (30) becomes

1

$$f_{dgf}(Y_{1\infty} - Y_{2\infty}) + (f_{init} + \varepsilon m_o)(Y_{3\infty} - Y_{4\infty}) + \frac{g \sin \theta_{o\infty}}{2} m_o(Y_{3\infty} + Y_{4\infty}) = 0$$
(35)

Notice that in the end of the hand-over process, the receiver's tangential forces at equilibrium (33), (34) correspond to those achieved for the one-hand case (as described at the end of Section II) while the giver's tangential forces at equilibrium correspond to a gravity-free grasp.

## B. Stability Analysis

To facilitate the analysis, we assume  $f_{dr}(t) = f_{init}$  and rewrite the closed loop system equation (5), (7) - (13) in the following compact form collecting all Lagrange multipliers in the vector  $\boldsymbol{\lambda} = [f_1 \ f_2 \ f_3 \ f_4 \ \lambda_1 \ \lambda_2 \ \lambda_3 \ \lambda_4]^T$  and all system position variables in  $\mathbf{x} = [\mathbf{q_1}^T \ \mathbf{q_2}^T \ \mathbf{q_3}^T \ \mathbf{q_4}^T \ \mathbf{p_o}^T \ \theta_o]^T$ .

$$M_{s}\ddot{\mathbf{x}} + C_{s}\dot{\mathbf{x}} + K_{v}\dot{\mathbf{x}} + A\boldsymbol{\lambda} - \begin{bmatrix} \frac{f_{dg}}{2r}J_{v1}^{T}(\mathbf{p_{t_{2}}} - \mathbf{p_{t_{1}}}) \\ -\frac{f_{dg}}{2r}J_{v2}^{T}(\mathbf{p_{t_{2}}} - \mathbf{p_{t_{1}}}) \\ \frac{f_{dr}}{2r}J_{v3}^{T}(\mathbf{p_{t_{4}}} - \mathbf{p_{t_{3}}}) \\ -\frac{f_{dr}}{2r}J_{v4}^{T}(\mathbf{p_{t_{4}}} - \mathbf{p_{t_{3}}}) \\ -\frac{g_{dr}}{2r}J_{v4}^{T}(\mathbf{p_{t_{4}}} - \mathbf{p_{t_{3}}}) \\ 0_{3\times1} \end{bmatrix} + r \begin{bmatrix} \hat{N}_{1}J_{\omega_{1}}^{T} \\ \hat{N}_{2}J_{\omega_{2}}^{T} \\ \hat{N}_{3}J_{\omega_{3}}^{T} \\ \hat{N}_{4}J_{\omega_{4}}^{T} \\ 0_{3\times1} \end{bmatrix} + \begin{bmatrix} -\frac{m_{gg}}{2}J_{v1}^{T} \\ -\frac{m_{gg}}{2}J_{v2}^{T} \\ -\frac{m_{gg}}{2}J_{v3}^{T} \\ -\frac{m_{gg}}{2}J_{v4}^{T} \\ 0 \\ m_{o}g \\ 0 \end{bmatrix} = 0 \quad (36)$$

with

$$M_{s} = \operatorname{diag} (M_{1}, M_{2}, M_{3}, M_{4}, M)$$

$$C_{s} = \operatorname{diag} (C_{1}, C_{2}, C_{3}, C_{4}, 0_{3 \times 3})$$

$$K_{v} = \operatorname{diag} (k_{v_{1}}I_{3}, k_{v_{2}}I_{3}, k_{v_{3}}I_{3}, k_{v_{4}}I_{3}, 0_{3 \times 3})$$

$$A = \begin{bmatrix} D & B \\ D_{o} & B_{o} \end{bmatrix}$$

$$D_{v} = K_{v} (D^{T}) - D_{v} = K_{v} (A^{T}) = 1$$

where  $D = \operatorname{diag}(D_{ii}^T)$ ,  $B = \operatorname{diag}(A_{ii}^T)$ ,  $i = 1, \dots, 4$ ,

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$$D_o = \begin{bmatrix} D_{15}^T & D_{25}^T & D_{35}^T & D_{45}^T & D_{55}^T \end{bmatrix}$$
$$B_o = \begin{bmatrix} A_{15}^T & A_{25}^T & A_{35}^T & A_{45}^T & A_{55}^T \end{bmatrix}$$

Similarly, the constraints can be written compactly as:  $A^T \dot{\mathbf{x}} = 0.$ 

Multiplying (36) by  $\dot{\mathbf{x}}^T$  from the left yields:  $\frac{dV}{dt} + W = 0$  where:

$$V = \frac{1}{2} \left( \dot{\mathbf{x}}^T M_s \dot{\mathbf{x}} + \sum_{i=1}^4 \gamma_i \hat{N}_i^2 + \gamma_M \Delta M_r^2 + \frac{f_{dg}}{2r} \| \mathbf{p_{t_1}} - \mathbf{p_{t_2}} \|^2 + \frac{f_{dr}}{2r} \| \mathbf{p_{t_3}} - \mathbf{p_{t_4}} \|^2 \right) + m_o g \Delta y + \frac{\Delta M_g g}{2} (\mathbf{p_{t_1}} + \mathbf{p_{t_2}})^T \begin{bmatrix} 0\\1 \end{bmatrix}$$
(37)

with  $\Delta M_g = m_o - m_g$ ,  $\Delta M_r = m_o - \hat{m}_o$ ,

$$\Delta y = \mathbf{p_o}^T \begin{bmatrix} 0\\1 \end{bmatrix} - \frac{1}{2} (\mathbf{p_{t_1}} + \mathbf{p_{t_2}} + \mathbf{p_{t_3}} + \mathbf{p_{t_4}})^T \begin{bmatrix} 0\\1 \end{bmatrix}$$

and

$$W = k_{v_1} \|\dot{\mathbf{q}}_1\|^2 + k_{v_2} \|\dot{\mathbf{q}}_2\|^2 + k_{v_3} \|\dot{\mathbf{q}}_3\|^2 + k_{v_4} \|\dot{\mathbf{q}}_4\|^2 - \frac{\dot{f}_{dg}}{4r} \|\mathbf{p_{t_1}} - \mathbf{p_{t_2}}\|^2 + \frac{\dot{m}_g g}{2} (\mathbf{p_{t_1}} + \mathbf{p_{t_2}})^T \begin{bmatrix} 0\\1 \end{bmatrix}$$
(38)

Similarly to [4], it is possible to prove that by appropriately choosing the control gains, (37) is locally positive definite in the constraint manifold defined by  $\mathcal{M}_c(\mathbf{x}) = {\mathbf{x} \in \mathbb{R}^{15} : A^T \dot{\mathbf{x}} = 0}.$ 

Considering  $|\dot{f}_{dg}| \leq b_1$  and  $|\dot{m}_g| \leq b_2$  for  $b_1, b_2 > 0, W$ can be bounded as follows:  $W \geq k_{v_1} ||\dot{\mathbf{q}}_1||^2 + k_{v_2} ||\dot{\mathbf{q}}_2||^2 + k_{v_3} ||\dot{\mathbf{q}}_3||^2 + k_{v_4} ||\dot{\mathbf{q}}_4||^2 - b_1 - b_2$ . Given that at the end of the hand-over process  $\dot{f}_{dg} = \dot{m}_g = 0$ , it is clear that  $\dot{V} = -W \leq 0$  and consequently  $V(t) \leq V(0)$  holds. The stability analysis follows a similar reasoning as in [4] to conclude that  $\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$  are bounded and converge to zero.

# **IV. SIMULATION RESULTS**

We consider two dual finger hands with identical robotic fingers, as depicted in Fig. 1, where r = 0.01 m and their parameters given in Table I. The receiver hand is placed at height h = 0.14 m above the giver hand while the fingers of each hand are positioned at distance d = 0.02 m and are initially at rest. We consider an object with parallel surfaces of width l = 0.02 m and height 1.8 \* l m with mass  $m_o =$ 0.08 Kg and  $I_o = 4 \times 10^{-4}$ . The hand-over timing parameters are given by  $t_0 = 5.5$  sec,  $t_{start} = 6$  sec and  $\Delta t = 0.5$  sec. The system position at time  $t_0$  is given in Table II. Giver and receiver control constants are  $k_{v_i} = 0.001$ ,  $\gamma_i = 0.001$ for i = 1, ..., 4,  $\gamma_M = 0.1$ ,  $\hat{m}_o(t_0) = 0.02$  kg,  $f_{init} = 0.5$ N,  $m_{qf} = 0$  kg,  $f_{dqf} = 0.1$  N and  $\varepsilon = 45$  as the proposed adaptation law (13) is used in the simulations. An initial grasp by the giver utilizing  $f_d = 4$  and  $\hat{m}_o(0) = 0.02$  kg precedes the hand-over process.

Simulation results depict system response in Fig. 3 - Fig. 10 including all stages of the hand-over process except the initial giver's grasp transients. Giver responses in Fig. 3 - Fig. 6 are depicted with dashed lines. Object load transfer (stage 3) occurs from t = 6 sec to t = 6.5 sec following stage 2. Notice the receiver's contact force and relative contact position transients at stage 2 (5.5 - 6 sec) in Fig. 3 and Fig.

4 respectively and the disturbance they induce to the giver's respective values which in turn stabilize at new values at the end of stage 2 as imposed by the equilibrium of the overall system. This is also evident by the change in the object's orientation (Fig. (7)). Also notice how during stage 3 the estimated mass of the receiver  $(\hat{m}_o)$  follows closely the released object mass  $(m_r)$  until the whole actual object mass at the end of the process (Fig. 5). When the object load transfer is over the system converges to a final equilibrium position (Fig. 4) corresponding to an almost inverse state from the one at the beginning of the object load transfer (Fig. 3 and 5); evidently, the different relative contact position at the final equilibrium (Fig. 4) means that contact forces are not exactly reversed. Force angles (Fig. 6) stay less than 6 degrees during all stages for both hands indicating that the object is securely delivered from the giver to the receiver hand avoiding slipping even under a narrow friction cone. Joint and object velocities converge to zero at the end of stage 2 and 3 indicating the new attained equilibriums (Fig. 8, 9); force and mass estimation errors depicted in Fig. 10 are also converging to zero at the end of each stage confirming theoretical findings.

Links	1	2	3
Masses (Kg)	0.045	0.03	0.015
Lengths (m)	0.04	0.03	0.02
Inertias (Kg m <sup>2</sup> )			
$I_z$ (×10 <sup>-6</sup> )	6	4	2

TABLE I: Robotic fingers parameters

Joints	$q_{i1}[deg]$	$q_{i2}[deg]$	$q_{i3}[deg]$
i = 1	150.725	-92.1141	-30.7137
i = 2	25.1163	66.9701	55.2918
i = 3	-131.626	53.7521	47.8739
i = 4	-35.8743	-52.597	-56.5287
Object	$x_o[m]$	$y_o[m]$	$\theta_o[deg]$
	0.0177	0.0635	0.0834

TABLE II: Initial system pose



Fig. 3: Contact force responses



Fig. 4: Relative contact positions  $Y_1 - Y_2$ ,  $Y_3 - Y_4$ 



Fig. 5: Object load transfer



Fig. 6: Force angles



Fig. 7: Object orientation



Fig. 8: Joint angular velocities of the hands

# V. CONCLUSIONS

A stable hand-over control scheme is proposed for two dual-fingered hands with semi-spherical fingertips under



Fig. 9: Object translational and angular velocities



Fig. 10: Force and mass estimation error responses. (a) Normal force error  $\Delta f_i$ . (b) Tangential force error  $\Delta \lambda_i$ . (c)  $\Delta N_i$  (d) Mass estimation error  $\Delta M$ 

contact and rolling constraints for the planar case. The control laws accomplish a natural object load transfer without utilizing any knowledge for the object mass and the contact locations. They are based on a controller achieving actual object mass estimate which is suitably modified. Grasp stability is achieved at each of the three stages of the handover process ensuring a secure object transfer. Simulation results illustrate the transients and achieved equilibriums at each stage.

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